

Chance-Constrained Target Tracking for Mobile Robots

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Abstract—This paper presents a robust target tracking algorithm for a mobile sensor with a fan-shaped field of view and finite sensing range. The goal of the mobile robot is to track a moving target such that the probability of losing the target is minimized. We assume that the distribution of the next position of a moving target can be estimated using a motion prediction algorithm. If the next position of a moving target has the Gaussian distribution, the proposed algorithm can guarantee the tracking success probability. In addition, the proposed method minimizes the moving distance of the mobile robot based on a bound on the tracking success probability. While the problem considered in this paper is a non-convex optimization problem, we derive analytical solutions which can be easily solved in real-time. The performance of the proposed method is evaluated extensively in simulation and validated in pedestrian following experiments using a Pioneer mobile robot with a Microsoft Kinect sensor.

I. INTRODUCTION

The problem of tracking a moving target has been actively studied for a number of applications such as hospital monitoring, surveillance, and museum guidance [1]. In general, a mobile sensor (or a mobile robot) is better for monitoring a large area over time than a static sensor whose sensing range is limited. In many cases, the mobile sensor has a bounded and limited field of view, e.g., laser range finders and RGB-D cameras. The bounded sensing region limits the ability to track a target. Hence, when we assume that a sensor is mounted on a robot, it is required to consider the orientation and range of the sensing region for controlling a robot. At the same time, it is desirable to reduce the moving distance of the robot to save energy. If a mobile robot is used in a domestic environment, such as hospitals, nursing homes, and homes, it is not desirable to draw attention from users by making frequent movements. Due to measurement noises and uncertainties in the environment, a mobile robot may fail to track a target with its finite sensing region. Hence, it is also important to guarantee the performance of a mobile robot against possible uncertainties.

The bounded sensing region is often not considered in target tracking and it is assumed that the position of a moving target can be measured at all times [2], [3]. Some authors have assumed sensors with a finite field of view. Bandyopadhyay et al. [4] defined a vantage zone, in which an observer could track a target, and proposed a control law

to minimize a risk function which is the time to escape this zone. Muppirala et al. [5] presented a motion strategy based on a critical event. A critical event signals the follower to perform rotational motion to prevent the target from escaping the sensing region. Masehian and Katebi [6] employed a parallel navigation law to reach a moving target using an omni-directional sensor.

In this paper, we are concerned with a sensor with a finite and fan-shaped field of view which is more suitable for many existing sensors, e.g., laser rangefinders, sonars, and RGB-D cameras. For those sensors, optimal control laws for minimizing a risk function have been proposed [7]–[9]. However, they assume a simple constant velocity model [9] and do not predict the motion of the target [7], [8]. Some authors proposed control inputs to minimize the uncertainty of the predicted location of the target [10], [11]. Instead, we focus on minimizing the tracking failure probability for the guaranteed performance. LaValle et al. [12] proposed a tracking algorithm for maximizing the probability of future visibility, which is similar to ours. However, their approach is applied to an omni-directional sensor and a solution is found by discretizing the search space. The proposed approach directly searches over the continuous search space with a directional sensor with a finite range and field of view.

This paper presents a motion strategy that maximizes tracking success probability. We apply the chance-constrained optimization method [13] to guarantee the upper bound on the tracking failure probability. The proposed approach provides robustness against uncertainties in our prediction about the target's next position. In addition, we minimize the moving distance of a robot. The problem is first formulated as a multi-objective optimization problem which minimizes both the upper bound on the tracking failure probability and the moving distance. We solve the problem sequentially by first finding a good upper bound on the tracking failure probability and then searching for a control which minimizes the moving distance while maintaining the upper bound on the tracking failure probability. While the proposed problem is non-convex, we derive a closed-form solution by finding solutions at different heading directions and choosing an optimal solution. We have validated the performance of the proposed method extensively from a number of simulations and experiments using a Pioneer robot.

The remainder of this paper is structured as follows. The target tracking problem is formulated in Section II. The dynamical model and sensing model are described in Section III and IV. An analytical solution to the proposed target tracking problem is presented in Section V and a nonparametric

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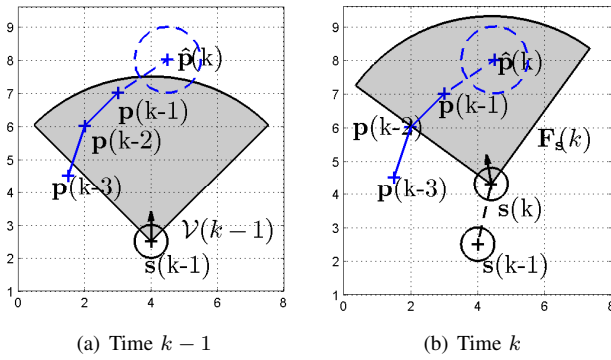


Fig. 1. An illustration of the target tracking problem considered in this paper. Gray regions are the sensing region of a mobile robot. A target is detected at time $k-1$ and is located at $\mathbf{p}(k-1)$. The predicted new location of the target is $\mathbf{p}(k)$ at time k . The blue dashed line represent the variance of the prediction. The mobile robot moves to $\mathbf{s}(k)$ to make sure that the target is within the sensing range with a guaranteed probability.

human motion prediction method is described in Section VI. Results from simulation and real-world experiments are presented in Section VII.

II. PROBLEM FORMULATION

We assume that a mobile robot and a target move on a 2D plane (see Figure 1). Let $\mathbf{s}(k) = [x_s(k) \ y_s(k)]^T$ be the position of the mobile robot at time k . The heading of the robot is the angle from the x -axis and denoted by $\phi_s(k)$. Let $\mathbf{u}(k)$ be the control input at time k . The motion of the robot is described by a discrete-time dynamic system

$$[\mathbf{s}(k+1) \ \phi_s(k+1)]^T = f(\mathbf{s}(k), \phi_s(k), \mathbf{u}(k)).$$

For the control $\mathbf{u}(k)$, the moving distance is denoted by $d(\mathbf{u}(k))$. The function f and $d(\mathbf{u}(k))$ are described in Section III.

We assume that the mobile robot carries a sensor with a finite and fan-shaped sensing region. The sensing region of the mobile robot at time k is denoted by $\mathcal{V}(k)$ (shaded regions in Figure 1). Its angular field of view is θ_s and the maximum range is R_s . We assume that the sensor is rigidly attached to the mobile robot, hence, its direction is the same as the heading of the robot.

Figure 1 shows an illustration, in which a mobile robot has detected a target from time $k-2$ to $k-1$ (Figure 1(a)) and moves to a new location to make sure the target is within robot's sensing range (Figure 1(b)). We assume that the distribution of the next position of the target is available using a motion prediction algorithm, such as Kalman filters or the autoregressive Gaussian process motion model [14]. Let $\mathbf{p}(k) = [x_T(k) \ y_T(k)]^T$ be the position of the target at time k . From the motion prediction algorithm, the target's position at time k using measurements up to time $k-1$ has the Gaussian distribution with mean $\hat{\mathbf{p}}(k)$ and covariance $\Sigma_T(k)$.

Our goal is to find control $\mathbf{u}(k-1)$, such that the target is within the sensing region of the mobile robot at time k . The deterministic visibility condition is $\mathbf{p}(k) \in \mathcal{V}(k)$. To consider the uncertainty in our prediction, we want the control to

guarantee the tracking failure probability, $P(\mathbf{p}(k) \notin \mathcal{V}(k))$. Using the chance-constrained optimization method [13], we formulate the tracking problem as the following multi-objective optimization problem:

$$\begin{aligned} \Pi_0 : \quad & \min_{\mathbf{u}(k-1)} \quad [\epsilon(k), d(\mathbf{u}(k-1))] \\ & \text{subject to} \quad P(\mathbf{p}(k) \notin \mathcal{V}(k)) \leq \epsilon(k), \\ & \quad \quad \quad 0 \leq E_1 \leq \epsilon(k) \leq E_2 \leq 1, \\ & \quad \quad \quad [\mathbf{s}(k) \ \phi_s(k)]^T \\ & \quad \quad \quad = f(\mathbf{s}(k-1), \phi_s(k-1), \mathbf{u}(k-1)), \end{aligned} \quad (1)$$

where $\epsilon(k)$ is the upper bound on the tracking failure probability. Here, our objective is to minimize the upper bound $\epsilon(k)$ instead of the tracking failure probability. Simultaneously, we aim to minimize the moving distance of a mobile sensor. Notice that the domain of $\epsilon(k)$ is a closed set $[E_1, E_2]$. Increasing E_1 may produce more tracking failures but with shorter moving distances. The value is a tuning parameter and its trade-off property is demonstrated in Section VII.

To solve this multi-objective optimization problem Π_0 , we optimize $\epsilon(k)$ before $d(\mathbf{u}(k-1))$ since successful tracking depends highly on $\epsilon(k)$. Thus, we design two subproblems: Π_1 and Π_2 . In Π_1 , we first determine $\epsilon(k)$ and $\phi(k)$. When the solution of Π_1 is given by $\epsilon^*(k)$ and $\phi^*(k)$, we solve Π_2 to find the optimal control $\mathbf{u}(k-1)$ by minimizing the moving distance $d(\mathbf{u}(k-1))$. These two subproblems are shown below:

$$\begin{aligned} \Pi_1 : \quad & \min_{\phi(k)} \quad \epsilon(k) \\ & \text{subject to} \quad \text{the feasible set in } \Pi_0 \text{ is non-empty.} \end{aligned} \quad (2)$$

$$\Pi_2 : \quad \min_{\mathbf{u}(k-1)} \quad d(\mathbf{u}(k-1)) \quad (3)$$

$$\begin{aligned} & \text{subject to} \quad P(\mathbf{p}(k) \notin \mathcal{V}(k)) \leq \epsilon^*(k), \\ & \quad \quad \quad [\mathbf{s}(k) \ \phi_s(k)]^T \\ & \quad \quad \quad = f(\mathbf{s}(k-1), \phi_s(k-1), \mathbf{u}(k-1)). \end{aligned} \quad (4)$$

III. MOBILE ROBOT'S DYNAMIC MODEL

A unicycle model is used to describe the dynamics of a mobile robot. The input control of the robot is denoted by $\mathbf{u} = [u_v \ u_w]^T$, where the directional velocity and the angular velocity are u_v and u_w , respectively. The admissible range of control inputs are:

$$V_{\min} \leq u_v \leq V_{\max} \quad \text{and} \quad W_{\min} \leq u_w \leq W_{\max}, \quad (5)$$

where $V_{\min}$, $V_{\max}$, $W_{\min}$, and $W_{\max}$ are determined by the vehicle. If u_v and u_w are constant from time $k-1$ to time k for a unit interval of length T , the state transition function can be written as:

$$\mathbf{s}(k) = \mathbf{s}(k-1) + \mathbf{f}_r(u_w)u_v, \quad (6)$$

where $\mathbf{f}_r(u_w) =$

$$\begin{cases} \begin{bmatrix} \frac{\sin \phi_s(k) - \sin \phi_s(k-1)}{u_w} \\ -\frac{\cos \phi_s(k) - \cos \phi_s(k-1)}{u_w} \end{bmatrix} & \text{if } u_w \neq 0, \\ \begin{bmatrix} T \cos \phi_s(k-1) \\ T \sin \phi_s(k-1) \end{bmatrix} & \text{if } u_w = 0. \end{cases}$$

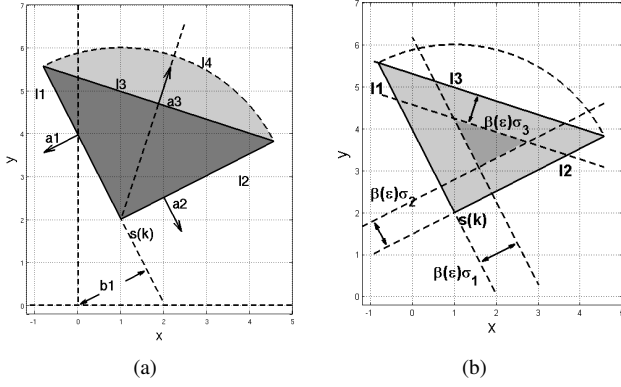


Fig. 2. (a) The light gray region is the sensing region of a mobile robot located at $\mathbf{s}(k)$. The dark gray region is the approximated sensing region. $\mathbf{a}_i$ is the normal vector of a line l_i . (b) The dark gray region is a desirable region to locate a target in order to minimize the tracking failure probability.

Its heading is updated from time $k-1$ to time k as:

$$\phi_s(k) = \phi_s(k-1) + u_w T.$$

The distance traveled by the robot can be found as follows:

$$d(\mathbf{u}) = \int_0^T \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = |u_v|T.$$

IV. BOUNDED FAN-SHAPED SENSING REGION

Our objective is to determine a one-step look-ahead motion strategy. Hence, we simplify notations by representing all variables and constraints relative to $\mathbf{s}(k-1)$.

Without loss of generality, we assume that $\mathbf{s}_0 = [0 \ 0]^T$ and $\phi_s(k-1) = \phi_0$, where $\mathbf{s}_0 = \mathbf{s}(k-1)$. Let $\mathbf{s} = [x_s \ y_s]^T$ be the position of the sensor at time k . The vector from $\mathbf{s}_0$ to the position of the target at time k is denoted by $\mathbf{p}$. The position $\mathbf{p}$ is distributed by a Gaussian distribution with mean $\hat{\mathbf{p}}$ and covariance $\Sigma_{\mathbf{T}}$. To simplify the notation further, we omit the time index k in our discussion below.

The sensing region at time k is a convex region bounded by l_1 , l_2 , and l_4 as shown in Figure 2(a). Since the tracking failure probability $P(\mathbf{p} \notin \mathcal{V})$ cannot be represented in a closed form, we approximate this sensing region by a triangle bounded by lines l_1 , l_2 and l_3 (the darker region in Figure 2(a)). Then, we define $\mathbf{a}_i$ as the normal vector of l_i and b_i as the shortest distance between l_i and the origin as follows:

$$\begin{aligned} \mathbf{a}_1 &= [-\sin(\phi_s + \theta_s/2) \ \cos(\phi_s + \theta_s/2)]^T, \\ \mathbf{a}_2 &= [\sin(\phi_s - \theta_s/2) \ -\cos(\phi_s - \theta_s/2)]^T, \\ \mathbf{a}_3 &= [\cos(\phi_s) \ \sin(\phi_s)]^T, \\ b_1 &= \mathbf{a}_1^T \mathbf{s}, \quad b_2 = \mathbf{a}_2^T \mathbf{s}, \\ b_3 &= \mathbf{a}_3^T \mathbf{s} + R_s \cos(\theta_s/2). \end{aligned}$$

We project $\mathbf{p}$ onto the normal vector $\mathbf{a}_i$. Then the projection $\mathbf{a}_i^T \mathbf{p}$ has the Gaussian distribution with mean $\mathbf{a}_i^T \hat{\mathbf{p}}$ and variance $\sigma_i^2 = \mathbf{a}_i^T \Sigma_{\mathbf{T}} \mathbf{a}_i$, since $\mathbf{p} \sim \mathcal{N}(\hat{\mathbf{p}}, \Sigma_{\mathbf{T}})$.

The following theorem shows the probabilistic visibility condition using the approximated sensing region.

Theorem 1: Given ϵ , suppose that $\mathbf{s}$ satisfies the following conditions:

$$\mathbf{a}_1^T \mathbf{s} \geq c_1 := \mathbf{a}_1^T \hat{\mathbf{p}} + \beta(\epsilon)\sigma_1, \quad (7)$$

$$\mathbf{a}_2^T \mathbf{s} \geq c_2 := \mathbf{a}_2^T \hat{\mathbf{p}} + \beta(\epsilon)\sigma_2, \text{ and} \quad (8)$$

$$\mathbf{a}_3^T \mathbf{s} \geq c_3 := \mathbf{a}_3^T \hat{\mathbf{p}} + \beta(\epsilon)\sigma_3 - R_s \cos(\theta_s/2), \quad (9)$$

where $\beta(\epsilon) = \Phi^{-1}(1 - \epsilon/3)$ and Φ is the cumulative distribution function of a standard normal random variable. Then, $P(\mathbf{p} \notin \mathcal{V}) \leq \epsilon$.

Proof: With respect to $\beta(\epsilon)$, the inequality (7) becomes

$$\frac{b_1 - \mathbf{a}_1^T \hat{\mathbf{p}}}{\sigma_1} \geq \beta(\epsilon).$$

Now we apply the function Φ to both sides to produce

$$P(\mathbf{a}_1^T \mathbf{p} \leq b_1) = \Phi\left(\frac{b_1 - \mathbf{a}_1^T \hat{\mathbf{p}}}{\sigma_1}\right) \geq 1 - \epsilon/3,$$

since $\beta(\epsilon) = \Phi^{-1}(1 - \epsilon/3)$ and Φ is a non-decreasing function. This is equivalent to $P(\mathbf{a}_1^T \mathbf{p} > b_1) \leq \epsilon/3$, because $\mathbf{a}_1^T \mathbf{p} \sim \mathcal{N}(\mathbf{a}_1^T \hat{\mathbf{p}}, \sigma_1^2)$. Similarly, we can derive $P(\mathbf{a}_i^T \mathbf{p} > b_i) \leq \epsilon/3$ for $i = 2$ and $i = 3$. Finally, we have

$$\begin{aligned} P(\mathbf{p} \notin \mathcal{V}) &\leq P\left(\bigcup_{i=1}^3 \{\mathbf{a}_i^T \mathbf{p} > b_i\}\right) \\ &\leq \sum_{i=1}^3 P(\mathbf{a}_i^T \mathbf{p} > b_i) \leq \epsilon. \end{aligned}$$

■

Intuitively, we can interpret the resulted constraints as a desirable region in which we want to locate a target to bound the tracking failure probability below ϵ . This reduced region is shown as the darker region in Figure 2(b).

V. MOTION STRATEGIES: ANALYTICAL SOLUTIONS

The optimization problem presented in the previous section has a non-convex domain since c_i is a nonlinear function of $\mathbf{u}$. We make the domain convex by fixing ϕ_s . For a fixed value of ϕ_s , we have a set of linear constraints and a solution can be easily found. Given a fixed ϕ_s , we first determine the optimal $\beta(\epsilon)$ by solving Π_1 and ϵ is computed using the cumulative distribution function of a standard normal random variable. With the computed $\beta(\epsilon)$, an optimal control can be determined by solving Π_2 . This procedure is repeated for each $\phi_s \in \mathcal{H}$, a set of candidate headings, to find the optimal control which minimizes the tracking failure probability and the moving distance.

A. Solutions to Π_1

Suppose that the optimization variable u_v is known such that $\phi_s = \phi_0 + u_w T \in [\phi_0 + W_{\min}T, \phi_0 + W_{\max}T]$. Now, u_v is the only remaining optimization variable. Given u_w , let $\mathbf{f}_r = \mathbf{f}_r(u_w)$, which is from (6). Then, each constraint $\mathbf{a}_i^T \mathbf{s} \geq c_i$ is equivalent to $(\mathbf{a}_i^T \mathbf{f}_r)u_v \geq c_i$. By combining these constraints with (5), the feasible set for u_v is

$$\Omega = \left\{ u_v \mid u_v \geq \max \left(V_{\min}, \max_{\mathbf{a}_i^T \mathbf{f}_r > 0} (c_i / (\mathbf{a}_i^T \mathbf{f}_r)) \right) \right\},$$

$$u_v \leq \min \left(V_{\max}, \min_{\mathbf{a}_i^T \mathbf{f}_r < 0} (c_i / (\mathbf{a}_i^T \mathbf{f}_r)) \right) \}. \quad (10)$$

To make Ω non-empty, the lower bound on u_v must be less than or equal to its upper bound. This new constraint can be represented as the following set of inequalities. For $i \in \{i | \mathbf{a}_i^T \mathbf{f}_r > 0\}$, $j \in \{j | \mathbf{a}_j^T \mathbf{f}_r < 0\}$, and $h \in \{h | \mathbf{a}_h^T \mathbf{f}_r = 0\}$,

$$c_i / (\mathbf{a}_i^T \mathbf{f}_r) \leq V_{\max}, \quad c_j / (\mathbf{a}_j^T \mathbf{f}_r) \geq V_{\min}, \quad (11)$$

$$c_i / (\mathbf{a}_i^T \mathbf{f}_r) \leq (c_j / (\mathbf{a}_j^T \mathbf{f}_r)), \quad c_h \leq 0. \quad (12)$$

With these constraints, the problem Π_1 can be reformulated as:

$$\max \quad \beta$$

subject to

$$\beta \leq \frac{V_{\max} \mathbf{a}_i^T \mathbf{f}_r - \bar{c}_i}{\sigma_i}, \quad \beta \leq \frac{V_{\min} \mathbf{a}_j^T \mathbf{f}_r - \bar{c}_j}{\sigma_j} \quad (13)$$

$$\beta \leq \frac{\bar{c}_i \mathbf{a}_j^T \mathbf{f}_r - \bar{c}_j \mathbf{a}_i^T \mathbf{f}_r}{\sigma_j \mathbf{a}_i^T \mathbf{f}_r - \sigma_i \mathbf{a}_j^T \mathbf{f}_r}, \quad \beta \leq -\frac{\bar{c}_h}{\sigma_h} \quad (14)$$

$$\Phi^{-1}(1 - E_2/3) \leq \beta \leq \Phi^{-1}(1 - E_1/3), \quad (15)$$

for $i \in \{i | \mathbf{a}_i^T \mathbf{f}_r > 0\}$, $j \in \{j | \mathbf{a}_j^T \mathbf{f}_r < 0\}$, and $h \in \{h | \mathbf{a}_h^T \mathbf{f}_r = 0\}$, where

$$\bar{c}_1 = \mathbf{a}_1^T \hat{\mathbf{p}}, \quad \bar{c}_2 = \mathbf{a}_2^T \hat{\mathbf{p}}, \quad \bar{c}_3 = \mathbf{a}_3^T \hat{\mathbf{p}} - R_s \cos(\theta_s/2).$$

Constraints (13)–(14) for β are equivalent to constraints (11)–(12). Then the problem is a convex problem with linear inequalities, which can be solved easily. The solution which maximizes β can be used to determine the minimum feasible ϵ .

We repeat this process and find the optimal $\epsilon(\phi_s)$ for each ϕ_s in $\mathcal{H}$. Then the solution to Π_1 is $\epsilon^* = \min\{\epsilon(\phi_s) : \phi_s \in \mathcal{H}\}$. The associated ϕ_s^* is used to determine $\mathbf{u}$.

B. Solutions to Π_2

Since the objective function $d(\mathbf{u})$ is an increasing function of u_v , the minimum feasible u_v is the optimal solution. Hence, by (10), the optimal u_v^* is

$$u_v^* = \max \left(V_{\min}, \max_{\mathbf{a}_i^T \mathbf{f}_r > 0} (c_i / (\mathbf{a}_i^T \mathbf{f}_r)) \right).$$

Finally, the optimal control is $\mathbf{u}^* = [u_v^* \ u_w^*]^T$, where u_w^* is determined by ϕ_s^* . The associated tracking guarantee is ϵ^* , which bounds the tracking failure rate.

VI. HUMAN MOTION PREDICTION

In this section, we focus on the problem of predicting the future trajectory of a moving person given current and past observations. We employ an autoregressive Gaussian process motion model (AR-GPMM) which was proposed in our previous work [14]. The model generates mean and variance of a predicted position of the target.

A. Autoregressive Gaussian Process Motion Model

We define a motion model as a mapping function $\mathcal{F}$ which maps recent m positions at time $k-1$, $\tau_{k-1} = \{(x, y)_i | i = k-1, \dots, k-m\}$, to the next position at time k , $(x, y)_k$, i.e., $\mathcal{F} : \mathbb{R}^{2m} \rightarrow \mathbb{R}^2$.

An autoregressive (AR) model is a way of representing a time-varying random process and an AR model of order m is defined as

$$X_k = c + \sum_{i=1}^m \psi_i X_{k-i} + w_{k-1}, \quad (16)$$

where $\psi_1, \dots, \psi_m$ are the parameters of AR model, c is a constant, and w_k is a process noise. As (16) is a linear model, if we assume the process noise w_k to be white Gaussian noise, the parameters can be estimated using the ordinary least-square method. Due to its simplicity, a linear dynamic model has been used often in practice. However, when it comes to real human motion prediction in practice, a linear model suffers due to its vulnerability to noises.

To handle these issues, Choi et al. [14] proposed an autoregressive Gaussian process motion model defined as follows:

$$X_k = f(X_{k-1}, \dots, X_{k-m}) \quad (17)$$

$$\sim GP_f(X_{k-1}, \dots, X_{k-m}), \quad (18)$$

where GP_f is a Gaussian process. The core of an AR-GPMM is Gaussian process regression which is a non-parametric Bayesian regression method, thus, it is expected to perform better on real-world environments. The performance of AR-GPMM is compared to a linear model in Section VII-B and VII-C.

VII. EXPERIMENTS

In order to show the validity and properties of the proposed motion strategy, we have conducted an extensive set of simulations in Sections VII-A and VII-B. Furthermore, we have validated the applicability by developing a human following robot to track a moving pedestrian.

A. Effects of the Prediction Covariance

This section describes the effects of Σ_T on the proposed method. In simulation, we show the relationship between the theoretical upper bound ϵ and the covariance. In addition, we show that the control depends on the shape of the covariance matrix. We assume the following parameterized covariance matrix:

$$\Sigma_T = \begin{bmatrix} \sigma_x^2 & \rho \sigma_x \sigma_y \\ \rho \sigma_x \sigma_y & \sigma_y^2 \end{bmatrix},$$

where σ_x^2 and σ_y^2 are variances of x_T and y_T , respectively, and their correlation coefficient is ρ . The current position of the mobile sensor is set to $[-300, -300]^T$ and its heading is $\pi/4$. The mean prediction of a target is $\hat{\mathbf{p}} = [100, 100]^T$.

For the purpose of this simulation study, we adopt weak dynamic constraints. The admissible range of control inputs is set to $-2000 \leq u_v \leq 2000$ and $-\pi/2 \leq u_w \leq \pi/2$. The sensing region is set to $\theta_s = 57^\circ$ and $R_s = 3500$ mm, which

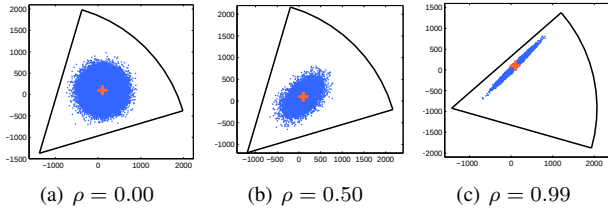


Fig. 3. Controls computed by the algorithm for different covariance shapes. Sampled target positions (blue dots), the sensing region (bounded by black solid lines), and the predicted position of the target (a red plus mark). Parameters for the covariance matrix are $\sigma_x = \sigma_y = 200\text{mm}$. For a small correlation, the target is located at the center of the sensing region.

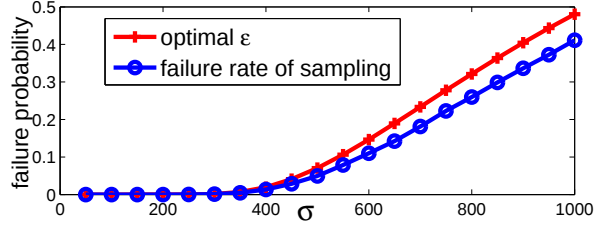


Fig. 4. Optimal upper bounds on the tracking failure probability and the tracking failure rate from samples at different variance values.

the Xbox software¹ requires. The setup is shown in Figure 3(a). Blue dots are 10^5 possible positions of a target sampled from the Gaussian distribution with mean $\hat{\mathbf{p}}$ and covariance Σ_T . We define the tracking failure probability of a control as a ratio of samples out of a sensing region to all samples.

Figure 3 shows the relationship between the control and the shape of the covariance matrix. When the absolute value of a correlation coefficient is small, the target is located in the center of the sensing region in Figures 3(a) and 3(b). The target with a large correlation coefficient is located near the edge of the sensing region, because the edge of an approximated sensing region can contain more distribution.

We also investigate the scenario where ρ is set to zero and σ increases from 50mm to 1000mm where $\sigma_x = \sigma_y = \sigma$. In Figure 4, the optimal upper bound ϵ and sampled tracking failure probability are plotted. As expected, the upper bound increases as variance σ^2 increases. In addition, we verify that the proposed algorithm guarantees the tracking failure probability below ϵ .

B. Target Tracking Using Real Human Trajectories

In this section, we validate our motion strategy for multi-step target tracking in simulation. In simulation, trajectories of a target are real human trajectories collected using the Vicon motion capture system. The length of a trajectory is set to 200 and there are 86 trajectories. We assume that there are measurement noises which are independently distributed as $\mathcal{N}(0, \sigma_n^2)$. We run 20 independent simulations for each trajectory with random measurement noises.

We have evaluated our motion strategy with two prediction algorithms: a Kalman filter with a linear model and AR-GPMM. The dynamic model is set to $-700 \leq u_v \leq 700$

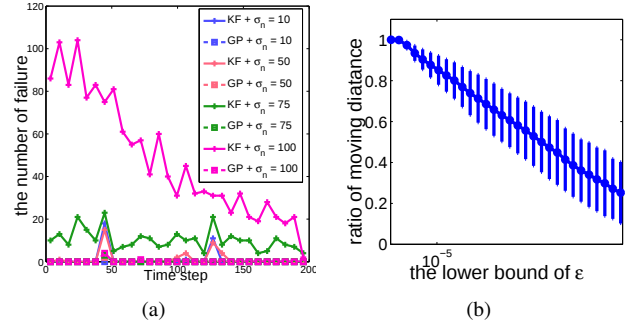


Fig. 5. (a) Histograms of tracking failures from the simulation using human trajectories at different noise levels. Histograms are plotted along the time step of a trajectory. (b) The moving distance as a function of E_1 . The moving distance ratio is computed by comparing the moving distance to the case when $E_1 = 0$. The average and standard deviation for 86 trajectories are plotted.

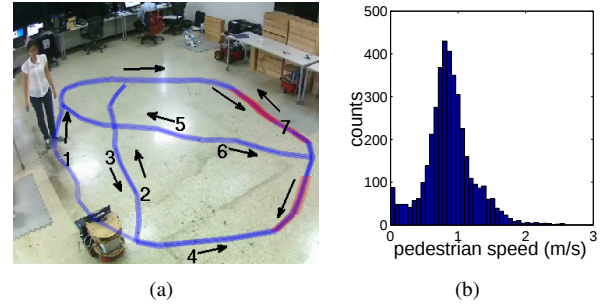


Fig. 6. (a) The setup for experiments in Section VII-C. The blue line and numbered arrows shows the path of a person. Red regions are the places where the robot lost a track due to the cluttered background. (b) A histogram of the pedestrian walking speed when a mobile robot successfully tracks a target.

and $-7\pi/9 \leq u_w \leq 7\pi/9$, according to the configuration of a Pioneer 3-AT mobile robot. The candidate set $\mathcal{H}$ has 51 uniformly spaced discrete values from $\phi_s(k) + W_{\min}T$ to $\phi_s(k) + W_{\max}T$ at time k . The range of ϵ is $E_1 = 0$ and $E_2 = 1$.

Figure 5(a) shows the number of tracking failures by different algorithms with different measurement noise levels at each time step of a trajectory. Interestingly, as variance of noises increases, the performance of our strategy with AR-GPMM is extremely good compared to the Kalman filter prediction. This is due to the fact that AR-GPMM is more robust against measurement noises.

We have also analyzed the relationship between the lower bound E_1 on ϵ and the moving distance (see Figure 5(b)). Increasing E_1 allows more candidate pairs of ϕ_s and ϵ , resulting in a shorter moving distance. While it reduces the moving distance, it also increases the tracking failure rate. Hence, there is a trade-off between the tracking failure probability and the moving distance.

C. Pedestrian Tracking Experiments

We used a Pioneer 3-AT and a Pioneer 3-DX mobile robot with a Microsoft Kinect camera mounted on top of the robot. We developed the algorithm in MATLAB. The position of a person is detected using the skeleton grab API of Xbox

¹<http://msdn.microsoft.com/en-us/library/hh973074.aspx>

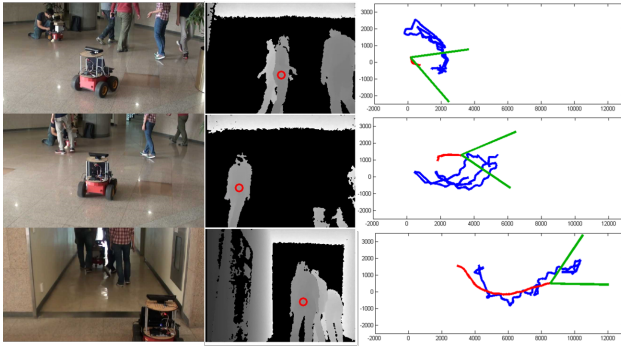


Fig. 7. Snapshots from the lobby experiment (first column). The second column shows the depth map from Kinect. Blue and red lines in the last column are trajectories of a target and a sensor, respectively. A green cone represent the sensing region of a robot.

software. Based on the Kinect sensor the sensing region is set to $\theta_s = 50^\circ$ and $R_s = 3000$ mm.

First, we conducted experiments similar to the simulation described in Section VII-B. On a floor, we marked 120 waypoints with a length of 37.43m as shown in Figure 6(a). A person moves from one waypoint to another waypoint in 2 seconds. We have performed a total of 10 trials using two prediction methods: the Kalman filter and AR-GPMM. To generate the same walking pattern for all 10 trials, we slow down all the settings for the system. The pedestrian walks quite slow and is detected at every 0.13s. The proposed tracking algorithm was executed at every 0.3s. Out of 10 trials, the Kalman filter based prediction method was successful for only three cases. The AR-GPMM based prediction method combined with the proposed motion control was successful for eight cases, showing its robust performance in practice. In our experiments, tracking fails at red regions shown in Figure 6(a) due to the cluttered background, which increases the measurement noise. In spite of the noise, the motion strategy with AR-GPMM prediction performs better than the Kalman filter prediction based controller.

We then conducted experiments to measure trackable pedestrian speed. Figure 6(b) shows the histogram of the pedestrian speed for 4,021 steps. In this experiment, we ran our motion control at 5 Hz and it was enough to track a person successfully. Considering that we set the maximum speed of the robot to 0.4m/s for safety, the tracking performance was excellent.

We have also performed experiments in an open hall, garage, lobby, and cafeteria. Figures 7 shows snapshots from these experiments. In all cases, the proposed motion controller was successful at tracking a moving pedestrian. In Figure 7, the path of the robot is much shorter than that of the pedestrian. Results from these field experiments are included in our video submission.

VIII. CONCLUSION

In this paper, we have presented a motion strategy for tracking a moving target with a guaranteed tracking failure probability. At the same time, the proposed method minimizes the moving distance for the optimized upper bound

on the tracking failure probability. Even though the sensing region is nonlinear and the problem is multi-objective and non-convex, we have derived analytical solutions which can be solved in real-time. From simulations, we have analyzed the properties of the proposed method. The method is also validated in physical experiments using a Pioneer robot with the Kinect sensor to track a moving pedestrian at various places. A major limitation of the work is an assumption of large free space areas. Thus we will focus on considering obstacles in the space. In addition, another approach of the future work is to consider false alarms in a crowded environment.

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