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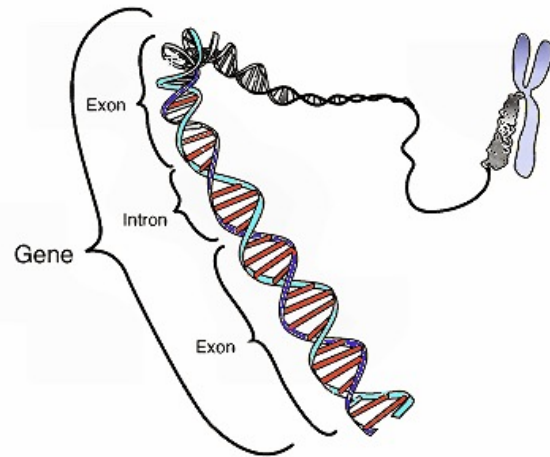
Introduction to Intelligent Systems

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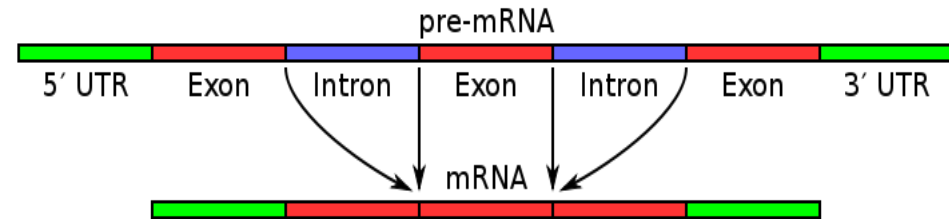
Ch. 14 Probabilistic Reasoning Over Time

DYNAMIC MODELS

Dynamic Model: An Example from Biology



[Source: Wikipedia]



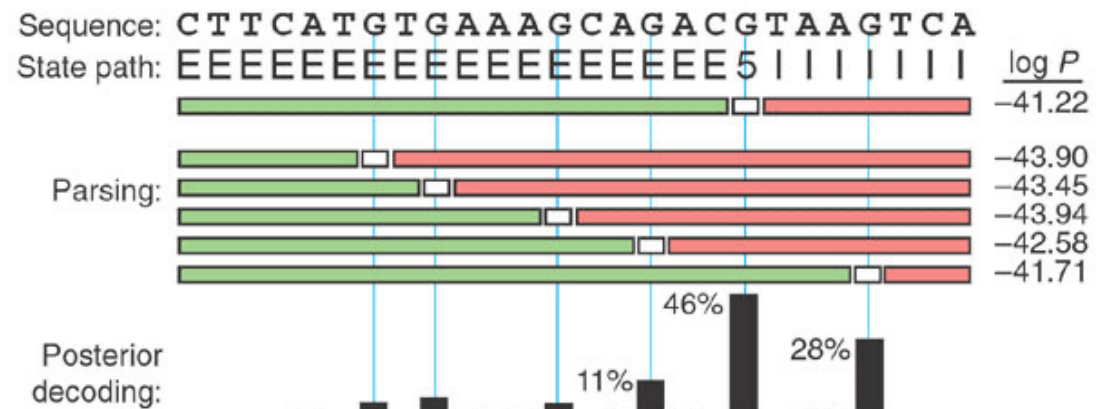
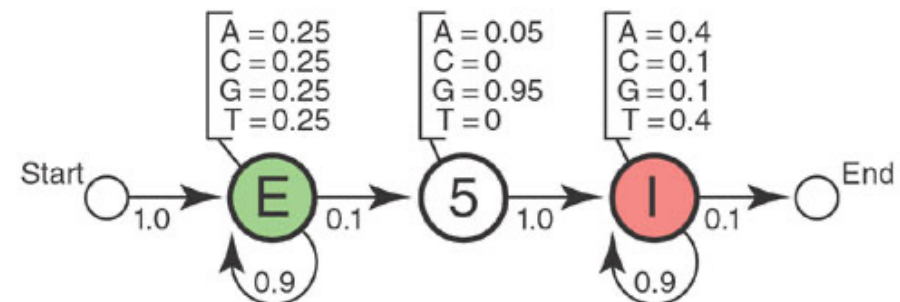
5' Splice-site recognition problem

To identify transitions from exon to intron

Toy hidden Markov model:

- Exons have a uniform base composition on average (25% each base)
- Introns are A/T rich (say, 40% each for A/T, 10% each for C/G)
- 5'SS consensus nucleotide is almost always a G (say, 95% G and 5% A)

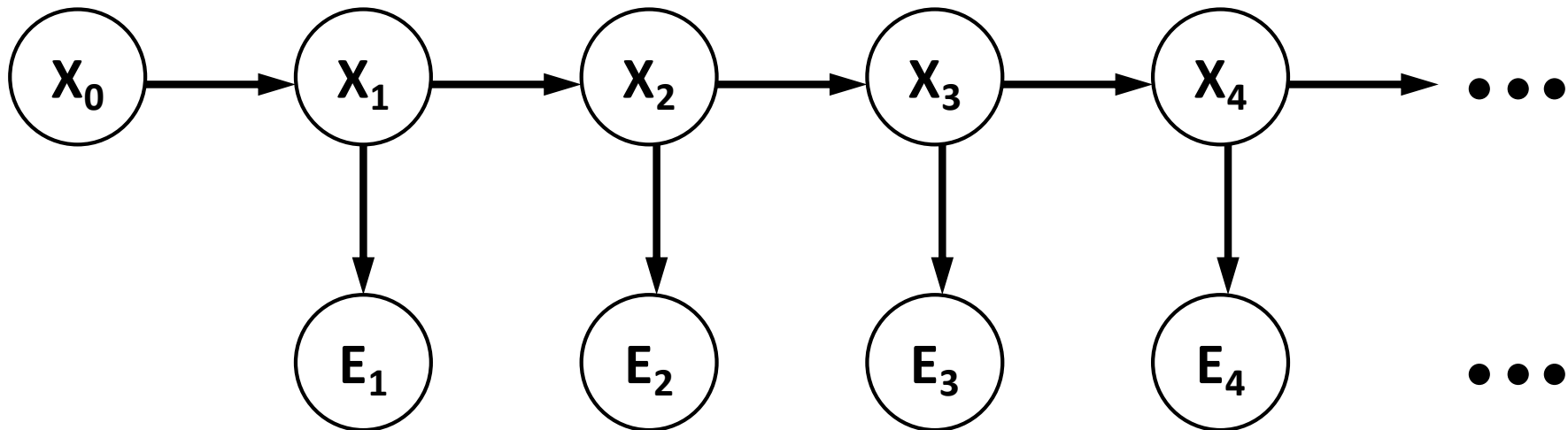
Hidden Markov Model (HMM)



Notations

- $\mathbf{X}_t$, state variables at time t
- $\mathbf{E}_t$, evidence variables at time t
- $\mathbf{e}_t$, actual observations at time t , i.e., a realization of $\mathbf{E}_t$.
- State evolution: $\mathbf{X}_0, \mathbf{X}_1, \mathbf{X}_2, \dots$
- Observation sequence: $\mathbf{E}_1, \mathbf{E}_2, \dots$
- $\mathbf{X}_{a:b} := \{\mathbf{X}_a, \mathbf{X}_{a+1}, \dots, \mathbf{X}_{b-1}, \mathbf{X}_b\}$

Bayesian Network



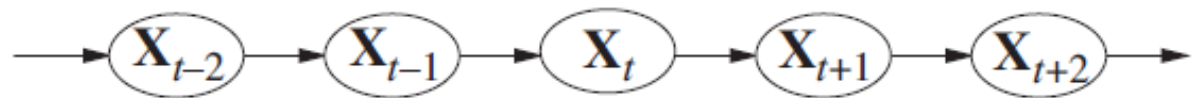
Transition Model

- **Transition model:** specifies the probability distribution over the latest state variables, given the previous values, i.e., $P(\mathbf{X}_t | \mathbf{X}_{0:t-1})$.
- **Markov assumption:** the current state depends on only a finite fixed number of previous states.
- **First-order Markov chain:** the current state depends only on the previous state and not on any earlier states, i.e.,

$$P(\mathbf{X}_t | \mathbf{X}_{0:t-1}) = P(\mathbf{X}_t | \mathbf{X}_{t-1})$$

- **Stationary process:** the probability distribution does not change when shifted over time, i.e., $P(\mathbf{X}_t | \mathbf{X}_{t-1})$ is the same for all t .

First-order Markov chain



Second-order Markov chain

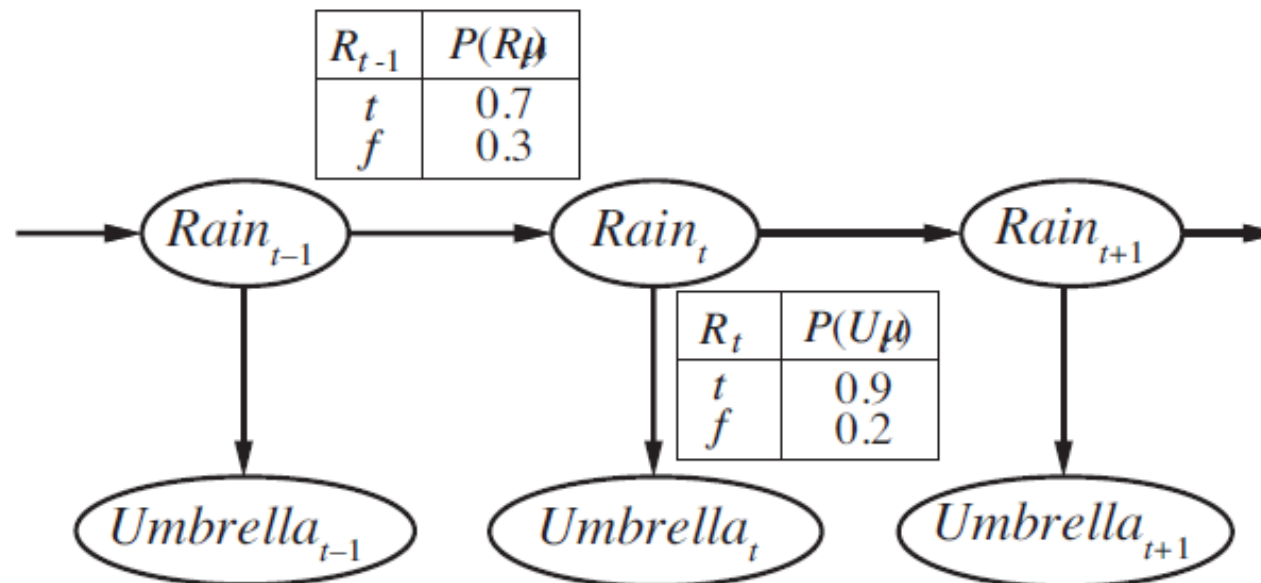


Sensor Model

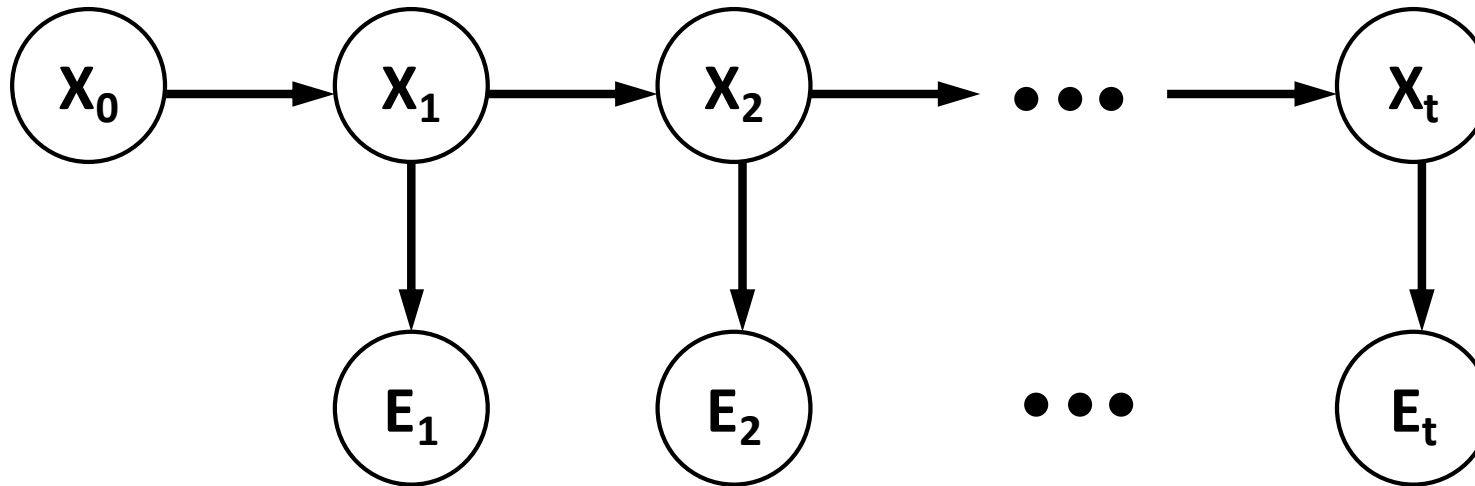
- **Sensor Markov assumption:** The evidence $\mathbf{E}_t$ depends only on the current state, i.e.,

$$P(\mathbf{E}_t | \mathbf{X}_{0:t}, \mathbf{E}_{1:t-1}) = P(\mathbf{E}_t | \mathbf{X}_t).$$

- **Observation model or sensor model** is $P(\mathbf{E}_t | \mathbf{X}_t)$.



Joint Distribution



$$\mathbf{P}(\mathbf{X}_{0:t}, \mathbf{E}_{1:t}) = \mathbf{P}(\mathbf{X}_0) \prod_{i=1}^t \mathbf{P}(\mathbf{X}_i | \mathbf{X}_{i-1}) \mathbf{P}(\mathbf{E}_i | \mathbf{X}_i)$$

Initial state model

transition model

sensor model