430.714

Estimation Theory

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INTRODUCTION

Estimation Problems

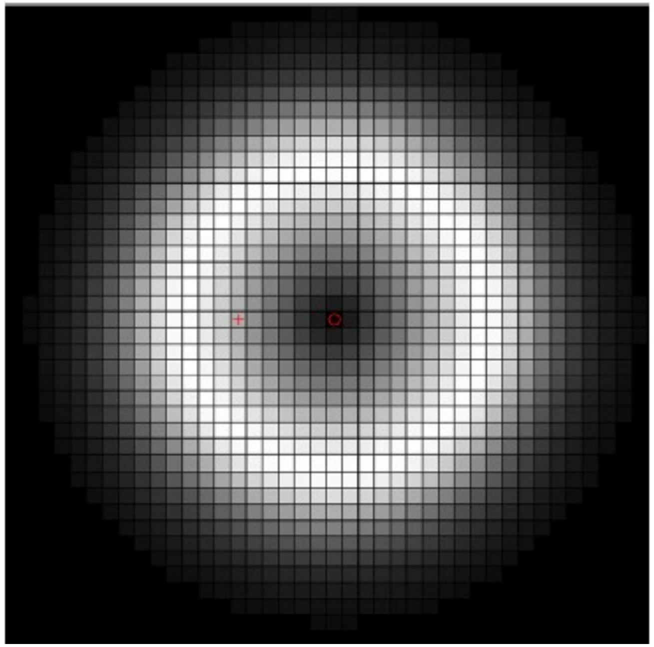
- Global positioning system (GPS)
- Signal processing – radar, sonar, speech, …
- Image analysis – lane detection, OCR, …
- Biomedicine – ultrasound, CT scan, MRI, …
- Communication – wireless phones, radio, …
- Control and robotics
- Seismology
- …

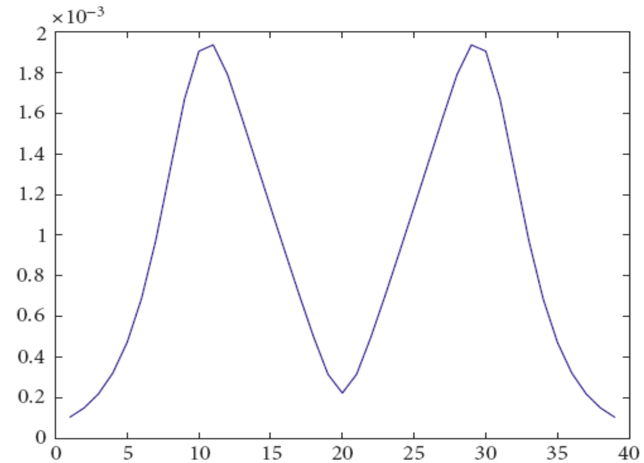
Example: Acoustic Amplitude Sensor

- Assumptions:
  - point source
  - lossless, isotropic propagation

$$z(x) = \frac{a}{\|x_{\text{source}} - x\|^{\frac{\alpha}{2}}} + w$$

- **z(x)**: root-mean-squared (rms) amplitude measurement at x
- **a**: amplitude at the sound source
- $\mathbf{x}_{\mathbf{source}}$: sound source position
- **x**: sensor position
- α: attenuation coefficient (acoustic: α=2)
- **w**: measurement noise (variance σ^2)





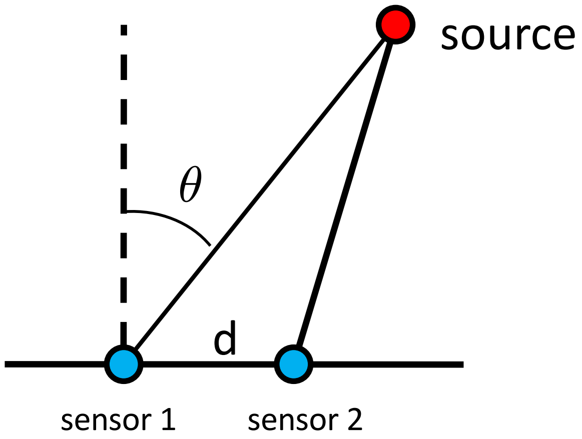
Example: Direction-Of-Arrival (DOA) Sensor

- Beamforming: M identical omnidirectional microphones

$$g_m(t) = s_0(t - t_m) + w_m(t)$$

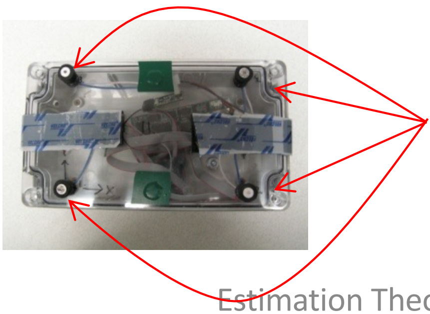
- $\mathbf{g_m}$: received signal at m-th microphone
- $\mathbf{s_0}$: source signal
- $\mathbf{w_m}$: noise
- $\mathbf{t_m}$: time delay

- If the source is far away, it can be considered as planar wave.

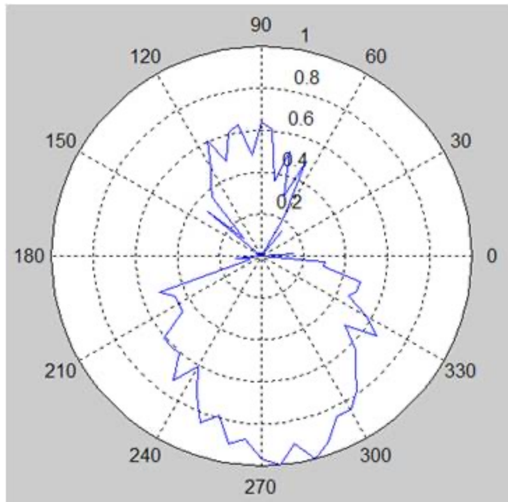


$$t_m = \frac{d}{c}\sin\theta$$

- **c**: speed of sound



microphones
10 cm x 6 cm



Questions

Given a problem

- What is a good representation of data?
- What is a good mathematical model?
- How to design a good estimator?
- How to measure performance of an estimator?

Assessing Estimator Performance

Model: $x[n] = A + w[n]$

- A: unknown parameter
- $w[n]$: zero mean white noise process (variance = σ^2)

Goal: From $\{x[0], x[1], \ldots, x[N-1]\}$, estimate A.

Estimator 1: $\hat{A}_1 = \frac{1}{N}\sum_{n=0}^{N-1} x[n]$

Estimator 2: $\hat{A}_2 = x[0]$

$$\begin{aligned} \mathbb{E}(\hat{A}_1) &= A \\ \mathbb{E}(\hat{A}_2) &= A \end{aligned}$$

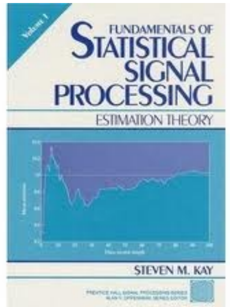
$$\begin{aligned} \textbf{var}(\hat{A}_1) &= \textbf{var}\left(\frac{1}{N}\sum_{n=0}^{N-1} x[n]\right) \\ &= \frac{1}{N^2}\sum_{n=0}^{N-1} \textbf{var}(x[n]) \\ &= \frac{\sigma^2}{N} \\ \textbf{var}(\hat{A}_2) &= \textbf{var}(x[0]) \\ &= \sigma^2 > \textbf{var}(\hat{A}_1) \end{aligned}$$

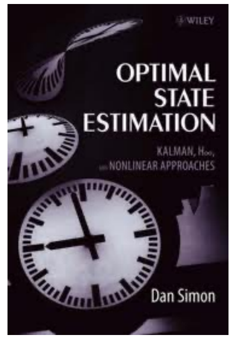
COURSE OUTLINE

Course Information

- Instructor: Songhwai Oh (songhwai@snu.ac.kr)
- TA: Yoseph Park (yoseph.park@rllab.snu.ac.kr)

- Textbooks (Recommended)
  - Steven M. Kay, "Fundamentals of Statistical Signal Processing, Volume I: Estimation Theory", Prentice Hall, 1993.
  - Dan Simon, "Optimal State Estimation: Kalman, H Infinity, and Nonlinear Approaches", Wiley-Interscience, 2006.





- Homepage:
  - http://rllab.snu.ac.kr/courses/estimation-theory-2026
- Class Board:
  - eTL

Topics

- Introduction and review of probability theory and linear algebra
- Minimum variance unbiased estimators
- Cramer-Rao bound
- Linear models and sufficient statistics
- Best linear unbiased estimators and maximum likelihood estimators
- Least squares, exponential family, and Bayesian approaches
- Multivariate Gaussian distribution
- Bayes risk, minimum mean square error (MMSE), and maximum a posteriori (MAP)
- Linear MMSE and sequential linear MMSE
- Bayesian filtering
- Kalman filtering
- Advanced topics in Kalman filtering
- Extended Kalman filter, unscented Kalman filter, and particle filter
- *Data association and multi-target tracking
- *Gaussian process regression (*if time permits)

Grading

- Class participation
- Homework
- Midterm (in class)
- Final exam (in class)