

430.714
Estimation Theory

Midterm

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Fall 2025

Student Name:

Student ID:

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IMPORTANT:

- Read all problems carefully and start with easier ones.
- You need to show all your steps for full credit.

Problem 1: [30 pts]

Consider the i.i.d. data $X_1, \dots, X_N$ from the Poisson distribution with parameter λ . Let $X = \{X_1, \dots, X_N\}$.

Recall that, if a random variable Z has the Poisson distribution, its probability mass function is

$$P(Z = k; \lambda) = e^{-\lambda} \frac{\lambda^k}{k!}.$$

- (a) Show that the regularity condition is satisfied for $P(X; \lambda)$.
- (b) Show that it is possible to find an efficient estimator of λ without computing the variance of an estimator.
- (c) Find the Cramer-Rao lower bound for λ .

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Problem 2: [30 pts]

Consider the following linear model

$$x[n] = A + Bn + w[n],$$

for $n = 0, 1, \dots, N - 1$, where $w[n]$ is a noise process with $\mathbb{E}(w[n]) = 0$ for all n . Let $r(n)$ be an autocorrelation function for $w[n]$, such that $r(n - m) = \mathbb{E}(w[n]w[m])$ for $0 \leq m, n < N$.

- (a) Find the best linear unbiased estimator of $[A, B]^T$.
- (b) Discuss when the best linear unbiased estimator is also the minimum variance unbiased estimator.
- (c) Now suppose that

$$x[n] = A + Bn + c[n] + w[n],$$

for $n = 0, 1, \dots, N - 1$, where $c[n]$ is a known sequence. Find the least squares estimator of $[A, B]^T$.

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Problem 3: [40 pts]

Suppose that T_i is the operation time of machine A at your factory between the $(i - 1)$ -th repair and the i -th failure. You think these operation times can be modeled well using the exponential distribution and want to find the average operation time.

Recall that an exponential random variable X with parameter $\lambda > 0$ has the following probability density function:

$$P(x \mid \lambda) = \lambda e^{-\lambda x},$$

for $x > 0$.

- (a) You model the operation time T_i as an exponential random variable with parameter λ and assume that T_i 's are independent. What is $\mathbb{E}(T_i)$?
- (b) Suppose that you have collected n data points, $D = \{t_1, \dots, t_n\}$. Find the maximum likelihood (ML) estimate for λ given D .
- (c) You have found a spec for the machine and assigned a prior distribution for λ according to the spec, such that λ has an exponential distribution with parameter $\beta > 0$. Find the maximum a posteriori (MAP) estimate for λ given D .
- (d) What is the average operation time of the machine? Discuss how the parameter β influences your estimate.

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