430.714

Estimation Theory

Prof. Songhwai Oh
ECE, SNU
INTRODUCTION
Estimation Problems

- Global positioning system (GPS)
- Signal processing – radar, sonar, speech, ...
- Image analysis – lane detection, OCR, ...
- Biomedicine – ultrasound, CT scan, MRI, ...
- Communication – wireless phones, radio, ...
- Control and robotics
- Seismology
- ...

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Estimation Theory
Example: Acoustic Amplitude Sensor

- **Assumptions:**
  - point source
  - lossless, isotropic propagation

\[ z(x) = \frac{a}{\| x_{\text{source}} - x \|} + w \]

- **\( z(x) \):** root-mean-squared (rms) amplitude measurement at \( x \)
- **\( a \):** amplitude at the sound source
- **\( x_{\text{source}} \):** sound source position
- **\( x \):** sensor position
- **\( \sim \):** attenuation coefficient (acoustic: \( \sim 2 \))
- **\( w \):** measurement noise (variance \( \sim 5 \))
Example: Direction-Of-Arrival (DOA) Sensor

• Beamforming: M identical omnidirectional microphones

\[ g_m(t) = s_0(t - t_m) + w_m(t) \]

  - \( g_m \): received signal at m-th microphone
  - \( s_0 \): source signal
  - \( w_m \): noise
  - \( t_m \): time delay

• If the source is far away, it can be considered as planar wave.

  \[ t_m = \frac{d}{c} \sin \theta \]

  - \( c \): speed of sound

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Estimation Theory
Questions

Given a problem
• What is a good representation of data?
• What is a good mathematical model?
• How to design a good estimator?
• How to measure performance of an estimator?
Assessing Estimator Performance

Model: \( x[n] = A + w[n] \)

- zero mean white noise process (variance = \(~^5\))
- unknown parameter

Goal: From \( \{x[0], x[1], \ldots, x[N - 1]\} \), estimate \( A \).

Estimator 1: \( \hat{A}_1 = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \)

\[ \text{var}(\hat{A}_1) = \text{var} \left( \frac{1}{N} \sum_{n=0}^{N-1} x[n] \right) = \frac{1}{N^2} \sum_{n=0}^{N-1} \text{var}(x[n]) = \frac{\sigma^2}{N} \]

\[ \mathbb{E}(\hat{A}_1) = A \]

Estimator 2: \( \hat{A}_2 = x[0] \)

\[ \text{var}(\hat{A}_2) = \text{var}(x[0]) = \sigma^2 > \text{var}(\hat{A}_1) \]
COURSE OUTLINE
Course Information

- Instructor: Songhwai Oh (songhwai@snu.ac.kr)
- TA: Timothy Ha (timothy.ha@rllab.snu.ac.kr)

- Textbooks (Recommended)

- Homepage:

- Class Board:
  - eTL
Topics

• Introduction and review of probability theory and linear algebra
• Minimum variance unbiased estimators
• Cramer-Rao bound
• Linear models and sufficient statistics
• Best linear unbiased estimators and maximum likelihood estimators
• Least squares, exponential family, and Bayesian approaches
• Multivariate Gaussian distribution
• Bayes risk, minimum mean square error (MMSE), and maximum a posteriori (MAP)
• Linear MMSE and sequential linear MMSE
• Bayesian filtering
• Kalman filtering
• Advanced topics in Kalman filtering
• Extended Kalman filter, unscented Kalman filter, and particle filter
• *Data association and multi-target tracking
• *Gaussian process regression (*if time permits)
Grading

• Class participation (online)
  – Recorded lectures in eTL
  – Q&A Session: Wed. 2:45-3:15 PM (Zoom)

• Homework (eTL)

• Midterm (in class)

• Final exam (in class)